

Nielsen Identity and the Renormalization Group Functions in an Abelian Supersymmetric Chern-Simons Model in the Superfield Formalism

A. G. Quinto* and A. F. Ferrari†

Universidade Federal do ABC - UFABC,

Rua Santa Adélia, 166, 09210-170, Santo André, SP, Brazil

Abstract

In this paper we study the Nielsen identity for the supersymmetric Chern-Simons-matter model in the superfield formalism, in three spacetime dimensions. The Nielsen identity is essential to understand the gauge invariance of the symmetry breaking mechanism, and it is obtained by using the BRST invariance of the model. We discuss the technical difficulties in applying this identity to the complete effective superpotential, but we show how we can study in detail the gauge independence of one part of the effective superpotential, K_{eff} . We calculate the renormalization group functions of the model for arbitrary gauge-fixing parameter, finding them to be independent of the gauge choice. This result can be used to argue that K_{eff} also does not depend on the gauge parameter. We discuss the possibility of the extension of these results to the complete effective superpotential.

*Electronic address: andres.quinto@ufabc.edu.br

†Electronic address: alysson.ferrari@ufabc.edu.br

I. INTRODUCTION

The effective potential is used to calculate physically meaningful quantities such as the masses for physical particles, and therefore its possible gauge dependence is an important question that have been studied in the literature for quite some time. Given that physical observables cannot depend on the gauge choice, it is essential to understand how to extract gauge independent information from perturbative calculations of the effective action in gauge theories [1–3]. A very robust formalism to address this question was developed by Nielsen, Kudo and Fukuda [4, 5], providing identities that encode the behavior of the effective action under changes of the gauge-fixing parameter. The so-called Nielsen identities imply that the gauge dependence of the effective action is compensated by a non-local field redefinition. For the effective potential, for example, the Nielsen identity reads

$$\left(\alpha \frac{\partial}{\partial \alpha} + C(\sigma; \alpha) \frac{\partial}{\partial \sigma} \right) V_{eff}(\sigma; \alpha) = 0, \quad (1)$$

where σ is the vacuum expectation value of the scalar field, α is the gauge-fixing parameter, V_{eff} the quantum effective potential, and $C(\sigma; \alpha)$ is a function which can be calculated in term of Feynman diagrams. A consequence of this relation is that physical quantities defined at extrema of the effective potential become gauge-independent [4, 6, 7]. We find in the literature many examples of the application of the Nielsen identities in condensed matter physics, QCD, QED, the Standard Model, ABJM theory, to name a few [8–14].

In the context of the Chern-Simons (CS) models, there have been reports of computations performed in a specific gauge, such as the evaluation of the renormalization group functions presented in [15, 16], which considered the models both with and without supersymmetry, assuming as true the gauge invariance. However, one must keep in mind that, on general grounds, renormalization group functions can depend on the choice of the gauge-fixing parameter [17–21]. Many other recent works in the literature also presented calculations in a specific gauge, without discussing the question of gauge dependence, such as studies regarding the effective superpotential of supersymmetric CS models coupled to matter [22–27], one exception being [28], which considered the large N limit. Since the effective potential can be dependent on the gauge-fixing parameter in general [1, 2], a proper study of gauge independence in CS models is still lacking.

Following theses ideas, our first goal is to study the Nielsen identity in the context of supersymmetric CS theory, working in the superfield formalism [29, 30], in which the su-

persymmetry is manifest in all stages of the calculation. We start by finding the general Becchi-Rouet-Stora-Tyutin (BRST) transformations associated to this theory, then use this result to obtain the Nielsen identity for the effective superpotential V_{eff}^S . The detailed development of the Nielsen formalism in the superfield language is the first result of this work. However, the direct application of this identity for superfield models in three spacetime dimensions is complicated by the difficulty in calculating the complete effective superpotential V_{eff}^S in the superfield language [24]. As a first step in this direction, we consider the part of the effective superpotential which does not depend on supercovariant derivatives of the background scalar superfield, K_{eff} . We calculate the renormalization group functions with arbitrary gauge-fixing parameter, and we show explicitly that these are indeed gauge independent. Since K_{eff} can be calculated from these functions as shown in [27], it follows that K_{eff} also does not depend on the gauge choice. We discuss how to extend these results to the complete effective superpotential V_{eff}^S .

This paper is organized as follows: in Section II, we present our model and study its invariance under BRST transformations. These results are used in Section III to find the Nielsen identity in the superfield formalism. The question of the gauge (in)dependence of the effective superpotential is discussed in Section (IV). In Section V, we calculate the renormalization group functions in the scale invariant version of our model, up to two loops, with an arbitrary gauge-fixing parameter. We find these functions to be gauge independent: this, together with the results of the other sections, allows us to firmly establish the gauge independence of part of the effective superpotential. Section VI presents our conclusions and perspectives. The two-loops integrals needed for our calculations are presented in the Appendix.

II. BUILDING AN INVARIANT LAGRANGIAN UNDER BRST TRANSFORMATIONS

In this section we investigate the BRST transformations in a $\mathcal{N} = 1$ supersymmetric Chern-Simons-matter model in $(2 + 1)$ dimensions. Our starting point is the action

$$\mathcal{S}_{CS} = \int d^5z \left\{ -\frac{1}{2} \Gamma^\alpha W_\alpha - \frac{1}{2} \overline{\nabla}^\alpha \Phi \nabla_\alpha \Phi - m \overline{\Phi} \Phi + \frac{\lambda}{4} (\overline{\Phi} \Phi)^2 \right\}, \quad (2)$$

where $W_\alpha = \frac{1}{2}D^\beta D_\alpha \Gamma_\beta$ is the gauge superfield strength, $\nabla^\alpha = (D^\alpha - ig \Gamma^\alpha)$ is the gauge supercovariant derivative, $D^\alpha = \partial^\alpha + i\theta_\beta \partial^{\beta\alpha}$ is the usual supersymmetric supercovariant derivative and m is a mass parameter. In this section we will work with a non vanishing mass parameter m (Higgs model), instead of a theory with conformal invariance at the classical level (Coleman-Weinberg [31] model), both for the sake of generality, and also because the $m = 0$ case would be more complicated to analyze in the context of the Nielsen identities [4, 6, 32].

The action 2 is invariant under the gauge transformations

$$\delta_g \Phi = ig \mathcal{K} \Phi, \quad \delta_g \bar{\Phi} = -ig \mathcal{K} \bar{\Phi}, \quad \delta_g \Gamma_\alpha = D_\alpha \mathcal{K}, \quad (3)$$

where \mathcal{K} is a scalar superfield; these can be rewritten as a BRST transformation,

$$\delta_B \Phi = i\epsilon g C \Phi, \quad \delta_B \bar{\Phi} = -i\epsilon g C \bar{\Phi}, \quad \delta_B \Gamma_\alpha = -\epsilon D_\alpha C, \quad (4)$$

where C is a ghost superfield and ϵ is an infinitesimal constant parameter, both being Grassmannian. As for the BRST transformation of the ghosts fields, since we consider an Abelian model [33], we have

$$\delta_B C = 0, \quad \delta_B \bar{C} = \epsilon B(z), \quad (5)$$

$B(z)$ being a scalar superfield, known as the Lautrup-Nakanishi auxiliary field in quantum field theory. The BRST transformations are nilpotent [7], i.e.,

$$\delta_B^2 C = \delta_B^2 \bar{C} = \delta_B^2 \Gamma_\alpha = \delta_B^2 \Phi = \delta_B^2 \bar{\Phi} = 0, \quad (6)$$

which implies that $\delta_B B(z) = 0$.

Now let us write the following Lagrangian,

$$\mathcal{L}_t = \mathcal{L}_{CSM} + \delta_B \mathcal{O}, \quad (7)$$

which is invariant by the BRST transformations; here,

$$\mathcal{O}(z) = \bar{C}(z) \left(-\alpha \frac{1}{4} B(z) + \frac{1}{2} F(z) \right), \quad (8)$$

where $F(z)$ is a gauge fixing function and α the gauge-fixing parameter. Applying the BRST transformation on the operator $\mathcal{O}(z)$, we have

$$\delta_B \mathcal{O} = -\epsilon \frac{\alpha}{4} B^2 + \epsilon \frac{1}{2} B F + \frac{1}{2} \bar{C} (\delta F). \quad (9)$$

Integrating out the superfield $B(z)$, we obtain

$$\mathcal{O}(z) = \frac{1}{4} \overline{C}(z) F(z), \quad (10)$$

and

$$\delta_B \mathcal{O} = \epsilon \frac{1}{4\alpha} F^2 + \frac{1}{2} \overline{C} (\delta F). \quad (11)$$

Therefore, the total Lagrangian, after setting $\epsilon = 1$, and redefining the scalar superfield as $\Phi = \frac{1}{\sqrt{2}} (\Phi_1 + i \Phi_2)$, reads

$$\begin{aligned} \mathcal{L}_t = & \frac{1}{2} \Gamma_\alpha W^\alpha + \frac{1}{2} (\Phi_1 D^2 \Phi_1 + \Phi_2 D^2 \Phi_2) - \frac{1}{2} m (\Phi_1^2 + \Phi_2^2) - \frac{1}{4} g^2 C^{\alpha\beta} \Gamma_\beta \Gamma_\alpha (\Phi_1^2 + \Phi_2^2) \\ & + \frac{1}{2} g [\Phi_1 D^\alpha \Phi_2 - \Phi_2 D^\alpha \Phi_1] \Gamma_\alpha + \frac{\lambda}{16} (\Phi_1^2 + \Phi_2^2)^2 + \frac{1}{4\alpha} F^2 + \frac{1}{2} \overline{C} \frac{\delta F}{\delta \mathcal{K}} C. \end{aligned} \quad (12)$$

An useful class of gauge fixing conditions, which is the supersymmetric generalization of the R_ξ gauge, is given by

$$F = D^\alpha \Gamma_\alpha + d g \Phi_2, \quad (13)$$

where d is an arbitrary parameter that can be chosen to eliminate the mixing between the Φ_2 and Γ_α , for example [22, 25]. We leave the value of d unspecified, in which case in general one would need to consider mixed propagators to evaluate quantum corrections [4, 6–8]. With this choice of gauge fixing, we end up with the Lagrangian

$$\begin{aligned} \mathcal{L}_t = & \frac{1}{4} \Gamma_\alpha W^\alpha + \frac{1}{2} (\Phi_1 [D^2 - m] \Phi_1 + \Phi_2 [D^2 - m] \Phi_2) \\ & - \frac{1}{4} g^2 C^{\alpha\beta} \Gamma_\beta \Gamma_\alpha (\Phi_1^2 + \Phi_2^2) + \frac{1}{2} g [\Phi_1 D^\alpha \Phi_2 - \Phi_2 D^\alpha \Phi_1] \Gamma_\alpha \\ & + \frac{\lambda}{16} (\Phi_1^2 + \Phi_2^2)^2 + \frac{1}{4\alpha} F^2 + \overline{C} D^2 C + \frac{1}{2} d g^2 \Phi_1 \overline{C} C, \end{aligned} \quad (14)$$

which is invariant under the BRST transformations:

$$\delta_B \Gamma_\alpha = -\epsilon D_\alpha C, \quad (15a)$$

$$\delta_B \Phi_1 = -\epsilon g \Phi_2 C, \quad (15b)$$

$$\delta_B \Phi_2 = \epsilon g \Phi_1 C, \quad (15c)$$

$$\delta_B \overline{C} = -\epsilon \frac{1}{\alpha} F, \quad (15d)$$

$$\delta_B C = 0. \quad (15e)$$

Finally, we add to the Lagrangian the source terms

$$\mathcal{L}_{source} = J^\mu \Gamma_\mu + \bar{\eta} C + \bar{C} \eta + f_1 \Phi_1 + f_2 \Phi_2 - g K_1 \Phi_2 C + g K_2 \Phi_1 C + h \mathcal{O}, \quad (16)$$

where J^μ , $\bar{\eta}$, η , f_1 and f_2 are the sources of the basic superfields, while K_1 , K_2 and h are sources for the composite operators.

III. OBTAINING THE NIELSEN IDENTITY

Our starting point is the generating functional,

$$Z[J_a] = e^{iW[J_a]} = N \int \mathcal{D}\phi_a e^{iS}, \quad (17)$$

where J_a represent all the sources and $\mathcal{D}\phi_a$ the path integral over all superfields that are present in the action

$$S = \int d^5z (\mathcal{L}_t + \mathcal{L}_{source}). \quad (18)$$

Applying the BRST transformations on $W[J_a]$, the invariance of \mathcal{L}_t implies that

$$0 = \frac{1}{Z[J_a]} N \int \mathcal{D}\phi_a e^{iS} \int d^5z (\delta_B \mathcal{L}_{source}), \quad (19)$$

where

$$\delta_B \mathcal{L}_{source} = \epsilon J^\mu D_\mu C - \epsilon \frac{1}{\alpha} F \eta - \epsilon g f_1 \Phi_2 C + \epsilon g f_2 \Phi_1 C + h \epsilon \tilde{\mathcal{O}}, \quad (20)$$

and

$$\delta_B \mathcal{O} = \epsilon \tilde{\mathcal{O}}. \quad (21)$$

We also quote the useful relations

$$\frac{\delta W[J_a]}{\delta J^\mu} = \Gamma_\mu, \quad \frac{\delta W[J_a]}{\delta \eta} = -\bar{C}, \quad (22a)$$

$$\frac{\delta W[J_a]}{\delta \bar{\eta}} = C, \quad \frac{\delta W[J_a]}{\delta f_1} = \Phi_1, \quad (22b)$$

$$\frac{\delta W[J_a]}{\delta f_2} = \Phi_2, \quad \frac{\delta W[J_a]}{\delta K_1} = -g \Phi_2 C, \quad (22c)$$

$$\frac{\delta W[J_a]}{\delta K_2} = g \Phi_1 C. \quad (22d)$$

The quantum effective action is defined by means of a partial Legendre transformation,

$$\Gamma[\phi_a] = W[J_a] - \int d^5z (J^\mu \Gamma_\mu + \bar{\eta} C + \bar{C} \eta + f_1 \Phi_1 + f_2 \Phi_2), \quad (23)$$

where the sources h and K_i are not Legendre transformed. Besides the usual relations,

$$\frac{\delta\Gamma[\phi_a]}{\delta\Gamma_\mu} = J^\mu, \quad \frac{\delta\Gamma[\phi_a]}{\delta\bar{C}} = \eta, \quad (24a)$$

$$\frac{\delta\Gamma[\phi_a]}{\delta C} = -\bar{\eta}, \quad \frac{\delta\Gamma[\phi_a]}{\delta\Phi_1} = -f_1, \quad (24b)$$

$$\frac{\delta\Gamma[\phi_a]}{\delta\Phi_2} = -f_2, \quad (24c)$$

one can also prove that [6],

$$\frac{\delta\Gamma[\phi_a]}{\delta h} = \frac{\delta W[J_a]}{\delta h}, \quad (25a)$$

$$\frac{\delta\Gamma[\phi_a]}{\delta K_i} = \frac{\delta W[J_a]}{\delta K_i}, \quad (25b)$$

$$\frac{\partial\Gamma[\phi_a]}{\partial\alpha} = \frac{\partial W[J_a]}{\partial\alpha}. \quad (25c)$$

Using these relations, Eq. (19) can be cast as

$$-\frac{1}{Z[J_a]} N \int \mathcal{D}\phi_a e^{iS} \int d^5z \left(h\tilde{\mathcal{O}} \right) = \int d^5z \left(\frac{\delta\Gamma[\phi_a]}{\delta\Gamma_\mu} D_\mu C - \frac{1}{\alpha} F \frac{\delta\Gamma[\phi_a]}{\delta\bar{C}} - \frac{\delta\Gamma[\phi_a]}{\delta\Phi_1} \frac{\delta\Gamma[\phi_a]}{\delta K_1} - \frac{\delta\Gamma[\phi_a]}{\delta\Phi_2} \frac{\delta\Gamma[\phi_a]}{\delta K_2} \right), \quad (26)$$

which after functional differentiation with respect to h and taking $h = 0$, reduces to

$$-\frac{1}{Z[J_a]} N \int \mathcal{D}\phi_a e^{iS} \tilde{\mathcal{O}} = \int d^5z \left(\frac{\delta\Gamma[\mathcal{O}(z)]}{\delta\Gamma_\mu} D_\mu C - \frac{1}{\alpha} F \frac{\delta\Gamma[\mathcal{O}(z)]}{\delta\bar{C}} - \frac{\delta\Gamma[\mathcal{O}(z)]}{\delta\Phi_1} \frac{\delta\Gamma[J_a]}{\delta K_1} - \frac{\delta\Gamma[\phi_a]}{\delta\Phi_1} \frac{\delta\Gamma[\mathcal{O}(z)]}{\delta K_1} - \frac{\delta\Gamma[\mathcal{O}(z)]}{\delta\Phi_2} \frac{\delta\Gamma[\phi_a]}{\delta K_2} - \frac{\delta\Gamma[\phi_a]}{\delta\Phi_2} \frac{\delta\Gamma[\mathcal{O}(z)]}{\delta K_2} \right), \quad (27)$$

where

$$\delta\Gamma[\mathcal{O}(z)] = \left. \frac{\delta\Gamma[\phi_a]}{\delta h} \right|_{h=0}. \quad (28)$$

The operator $\tilde{\mathcal{O}}$ in (27), in the class of supersymmetric R_ξ gauges we are considering, is given explicitly by

$$\tilde{\mathcal{O}} = \frac{1}{4\alpha} F^2 + \bar{C} D^2 C + \frac{1}{2} d g^2 \Phi_1 \bar{C} C, \quad (29)$$

which, by using the equation of motion $\delta S/\delta\bar{C} = 0$, reduces to

$$\tilde{\mathcal{O}} = \frac{1}{4\alpha} F^2 - \bar{C} \eta. \quad (30)$$

By differentiation of $W[J] = -i \ln Z[J]$ with respect to the gauge parameter α , considering Eq. (14), one obtains that

$$\alpha \frac{\partial W[J_a]}{\partial \alpha} = \frac{1}{Z[J_a]} N \int \mathcal{D}\phi_a \left(- \int d^5 z \frac{1}{4\alpha} F^2 \right) e^{iS}, \quad (31)$$

and proceeding similarly,

$$- \int d^5 r \eta(r) \frac{\delta W[J_a]}{\delta \eta(r)} = \frac{1}{Z[J_a]} N \int \mathcal{D}\phi_a e^{iS} \left(\int d^5 r \eta(r) \overline{C}(r) \right). \quad (32)$$

These relations, together with Eqs. (25), (24) and (22), allows us to rewrite Eq. (27) as

$$\begin{aligned} \alpha \frac{\partial \Gamma[\phi_a]}{\partial \alpha} + \int d^5 z \frac{\delta \Gamma[\phi_a]}{\delta \overline{C}} \overline{C} = & \int d^5 r \int d^5 z \left(\frac{\delta \Gamma[\mathcal{O}(z)]}{\delta \Gamma_\mu} D_\mu C - \frac{1}{\alpha} F \frac{\delta \Gamma[\mathcal{O}(z)]}{\delta \overline{C}} \right. \\ & - \frac{\delta \Gamma[\mathcal{O}(z)]}{\delta \Phi_1} \frac{\delta \Gamma[\phi_a]}{\delta K_1} - \frac{\delta \Gamma[\phi_a]}{\delta \Phi_1} \frac{\delta \Gamma[\mathcal{O}(z)]}{\delta K_1} - \frac{\delta \Gamma[\mathcal{O}(z)]}{\delta \Phi_2} \frac{\delta \Gamma[\phi_a]}{\delta K_2} \\ & \left. - \frac{\delta \Gamma[\phi_a]}{\delta \Phi_2} \frac{\delta \Gamma[\mathcal{O}(z)]}{\delta K_2} \right). \end{aligned} \quad (33)$$

This expression is the base for obtaining the Nielsen identity.

The effective superpotential is obtained by setting $\Phi_1 = \sigma_{cl}$ in the effective action,

$$\Gamma[\Phi_1, \alpha]|_{\Phi_1=\sigma_{cl}} = V_{eff}^S(\sigma_{cl}, \alpha), \quad (34)$$

where

$$\sigma_{cl} = \sigma_1 - \theta^2 \sigma_2 \quad (35)$$

is the spacetime constant expectation value of the scalar superfield. By taking $\overline{C} = C = \Phi_2 = \Gamma_\alpha = 0$ in Eq. (33), after some manipulations, we end up with

$$\left[\alpha \frac{\partial}{\partial \alpha} + C^S(\sigma_{cl}, \alpha) \frac{\partial}{\partial \sigma_{cl}} \right] V_{eff}^S(\sigma_{cl}, \alpha) = 0, \quad (36)$$

where

$$C^S(\sigma_{cl}, \alpha) = \int d^5 z \frac{\delta^2 \Gamma[\mathcal{O}(z)]}{\delta K_1(0) \delta h(y)} \Big|_{K_1=h=0}, \quad (37)$$

which is the Nielsen identity for the superpotential.

IV. ON THE GAUGE (IN)DEPENDENCE OF THE EFFECTIVE SUPERPOTENTIAL

The effective superpotential in three spacetime dimensions have the general form

$$V_{eff}^S(\sigma_{cl}, \alpha) = - \int d^5 z \left[K_{eff}(\sigma_{cl}, \alpha) + \mathcal{F}(D_\alpha \sigma_{cl}, D^\alpha \sigma_{cl}, D^2 \sigma_{cl}, \sigma_{cl}, \alpha) \right], \quad (38)$$

where we made explicit the potential gauge dependence. Here, K_{eff} is the part of the effective superpotential that do not depend on derivatives of the background classical superfield σ_{cl} , similarly to the Kählerian effective superpotential defined in four dimensional models [29, 30]. In the context of dynamical gauge symmetry breaking, it is enough to consider only the K_{eff} [22, 25–28], while a study of a possible supersymmetry breaking would involve also the knowledge of \mathcal{F} [34, 35].

An explicit perturbative evaluation of V_{eff}^S starting from Eq. (14), within the superfield formalism, is in general quite difficult. The root of this problem is the fact that the classical superfield σ_{cl} is spacetime constant, but its covariant derivatives do not vanish, $D_\alpha \sigma \neq 0$. This, for example, complicates the calculation of the free superpropagators of the model, since powers of σ_{cl} appear in the quadratic operators which have to be inverted. This leads to the appearance of noncovariant superpropagators, as shown in [34, 36]. In these works, the effective superpotential of the supersymmetric Chern-Simons-matter and QED models was calculated up to two loops. Part of these calculations was performed in an arbitrary gauge, but the final results for the effective superpotential are obtained in the Landau gauge. Another perspective on the difficulties of evaluating the full effective superpotential in the superfield formalism, using heat kernel techniques, can be found in [24].

One possibility to obtain a full computation of V_{eff}^S within the superfield formalism would involve the use of the Renormalization Group Equation (RGE). This approach was used to calculate K_{eff} in the massless limit of the model, in which case the action (2) is scale invariant, in [27]. Essentially, one may consider K_{eff} as a function of the single mass scale σ_1 , and imposes the RGE,

$$\left[\mu \frac{\partial}{\partial \mu} + \beta_x \frac{\partial}{\partial x} - \gamma_\Phi \sigma_1 \frac{\partial}{\partial \sigma_1} \right] K_{eff}(\sigma_1; \mu, x, L) = 0, \quad (39)$$

where x generically denotes the coupling constants of the theory, μ is the mass scale introduced by the regularization,

$$L = \ln \left[\frac{\sigma_1^2}{\mu} \right], \quad (40)$$

and γ_Φ is the anomalous dimension of scalar superfield. The scale invariance of the model constrains the form of the radiative corrections to K_{eff} , so that a simple ansatz can be made, which inserted in Eq. (39) provides a set of recursive equations from which coefficients of the so-called leading logs contributions to K_{eff} can be found. This technique could

be in principle extended for the full effective superpotential V_{eff}^S , but in this case more complicated, multiscale techniques would be needed [37, 38], since V_{eff}^S should be considered as a function both of σ_1 and σ_2 .

The derivation of the effective superpotential from the renormalization group functions, by means of the RGE, may allow one to infer from the gauge (in)dependence of the beta functions and anomalous dimensions the gauge (in)dependence of the effective superpotential itself. This is something we can do for the K_{eff} , since it was already established in [27] how it can be calculated from the renormalization group functions. Therefore, in the next section, we will present a detailed computation of the beta and gamma functions in an arbitrary gauge, showing that the result is indeed gauge independent. By this reasoning, we can conclude that K_{eff} do not depend on the gauge parameter. That means, when only K_{eff} is considered, the Nielsen identity (36) is trivially satisfied with $C^S = 0$.

These results may suggest the gauge independence of the whole effective superpotential V_{eff}^S , but without explicitly establishing that the RGE fixes the form of \mathcal{F} in some approximation, without ambiguities, from the renormalization group functions, which we know are gauge independent, we believe this is still an open question. The discussion of [34, 36] is not conclusive in this regard, since most of the results are presented in a specific gauge, but some gauge dependence was found in the effective superpotential of the supersymmetric QED model. It is also not simple to use the Nielsen identity itself to investigate this point since, as discussed in [4, 6], the calculation of C^S in the massless case is complicated by the fact that different loop orders contribute to C^S in a given order in the coupling constants.

V. GAUGE INVARIANCE OF THE RENORMALIZATION GROUP FUNCTIONS

In this section, to establish the gauge independence of the renormalization group functions of our model, we consider the $m = 0$ version of Eq. (2), generalized to exhibit a global $SU(N)$ symmetry,

$$\mathcal{S}_{CS} = \int d^5z \left\{ -\frac{1}{2} \Gamma^\alpha W_\alpha - \frac{1}{2} \overline{\nabla^\alpha \Phi_a} \nabla_\alpha \Phi_a + \frac{\lambda}{4} (\overline{\Phi_a} \Phi_a)^2 \right\}, \quad (41)$$

where the N matter superfields carry indices of the fundamental representation of the $SU(N)$ group. We introduce the gauge-fixing action,

$$\mathcal{S}_{GF} = \frac{1}{4\alpha} \int d^5z (D^\alpha \Gamma_\alpha)^2, \quad (42)$$

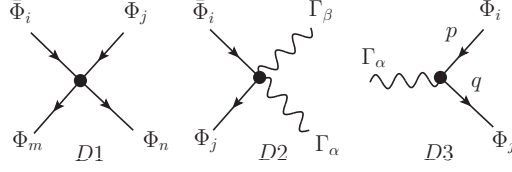


Figure 1: Elementary vertices of the model, where a continuous line is associated to a scalar superfield, and a wavy line to the gauge superfield.

but differently from what is done in the literature [15, 22, 28], we will perform all calculations with an arbitrary gauge-fixing parameter. From Eqs. (41) and (42) we have,

$$\mathcal{S} = \int d^5 z \left\{ \frac{1}{4} \Gamma_\alpha \left[D^\beta D^\alpha + \frac{1}{\alpha} D^\alpha D^\beta \right] \Gamma_\beta - \frac{1}{2} g^2 C_{\beta\alpha} \Gamma^\alpha \Gamma^\beta \bar{\Phi}_a \Phi_a \right. \\ \left. + \bar{\Phi}_a D^2 \Phi_a - \frac{i}{2} g \left[\Gamma^\alpha \bar{\Phi}_a D_\alpha \Phi_a - (D^\alpha \bar{\Phi}_a) \Gamma_\alpha \Phi_a \right] + \frac{\lambda}{4} (\bar{\Phi}_a \Phi_a)^2 + \mathcal{L}_{ct} \right\}, \quad (43)$$

where the counterterm Lagrangian is,

$$\mathcal{L}_{ct} = \frac{(Z_\Gamma - 1)}{4} \Gamma_\alpha D^\beta D^\alpha \Gamma_\beta + \frac{(Z_\Phi - 1)}{2} \bar{\nabla}^\alpha \bar{\Phi}_a \nabla_\alpha \Phi_a + \frac{\lambda}{4} Z_\lambda (\bar{\Phi}_a \Phi_a)^2, \quad (44)$$

Z_Γ , Z_Φ and Z_λ being the counterterms needed to make the renormalized quantities finite in each order of perturbation theory. From Eq. (43) it follows the scalar and gauge propagators,

$$\langle \bar{\Phi}_i(k, \theta_1) \Phi_j(-k, \theta_2) \rangle = i \delta_{ij} \frac{D^2}{k^2} \delta^2(\theta_1 - \theta_2), \quad (45)$$

and

$$\langle \Gamma_\beta(k, \theta_1) \Gamma_\rho(-k, \theta_2) \rangle = \frac{i}{2k^2} (D_\rho D_\beta + \alpha D_\beta D_\rho) \delta^2(\theta_1 - \theta_2), \quad (46)$$

where

$$D_\rho D_\beta + \alpha D_\beta D_\rho = b(\alpha) k_{\rho\beta} + a(\alpha) C_{\beta\alpha} D^2, \quad (47)$$

and

$$a(\alpha) = 1 - \alpha; \quad b(\alpha) \equiv 1 + \alpha. \quad (48)$$

The elementary vertices are represented in Figure 1 and their analytic expression are

$$i V_{(\bar{\Phi}_a \Phi_a)^2} = 2 i \lambda (\delta_{in} \delta_{jm} + \delta_{im} \delta_{jn}), \quad (49)$$

$$i V_{\bar{\Phi}_a \Phi_a \Gamma_\beta \Gamma_\alpha} = i g^2 \delta_{ij} C^{\alpha\beta}, \quad (50)$$

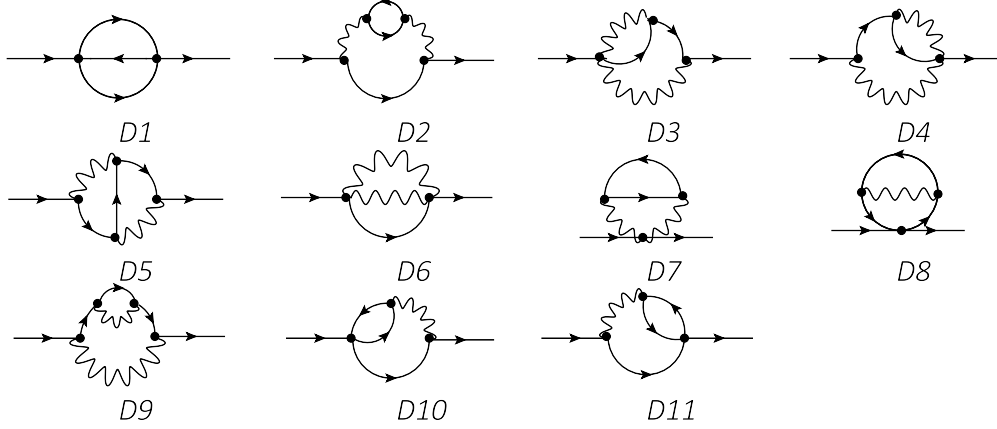


Figure 2: Two loops diagrams contributing to the two-point vertex function of the scalar superfield Φ .

and

$$i V_{\Phi_a D^\alpha \bar{\Phi}_a \Gamma_\alpha - \Phi_a D^\alpha \Phi_a \Gamma_\alpha} = \frac{1}{2} \delta_{ij} g [D^\alpha(p) - D^\alpha(-q)], \quad (51)$$

where an integration in the Grassmann coordinate of the superspace is omitted at each vertex.

The use of regularization by dimensional reduction means that all super-algebra manipulations are performed in three dimensions, while momentum integrals are calculated at dimension $d = 3 - \epsilon$. The use of this regularization scheme guarantees that the one loop correction are finite. All algebraic manipulations of supercovariant derivatives needed for the evaluation of supergraphs were performed with the Mathematica package SusyMath [39]; explicit details about the calculations will be presented elsewhere [40].

We start by calculating the two-point vertex functions associated to scalar and gauge superfields. For the case of the scalar superfield, the diagrams that contribute are represented in the Figure 2, and the corresponding results are given in Table I. For the divergent part of the two points vertex function, up to two loops, we can write

$$S_{\bar{\Phi}\Phi}^{2loop} = \frac{i}{4(32\pi^2\epsilon)} \left[-(N+1)\lambda^2 + \frac{1}{4}(a+b)^2(2N+3)g^4 \right] \int \frac{d^3p}{(2\pi)^3} d^2\theta \bar{\Phi}_i(p, \theta) D^2 \Phi_i(-p, \theta), \quad (52)$$

$D1$	$-2(N+1)\lambda^2$	$D2$	$(a+b)^2 g^4 N$	$D3$	$a(b-3a)g^4$	$D4$	$a(b-3a)g^4$
$D5$	$\frac{1}{2}(3a^2+2ab+3b^2)g^4$	$D6$	$2a^2g^4$	$D7$	0	$D8$	0
$D9$	$4a^2g^4$	$D10$	0	$D11$	0		

Table I: Divergent contributions from each diagram presented in Figure 2, with the common factor

$$\frac{1}{8} \left(\frac{i}{32\pi^2\epsilon} \right) \int \frac{d^3p}{(2\pi)^3} d^2\theta \bar{\Phi}_i(p, \theta) D^2 \Phi_i(-p, \theta) \text{ omitted.}$$

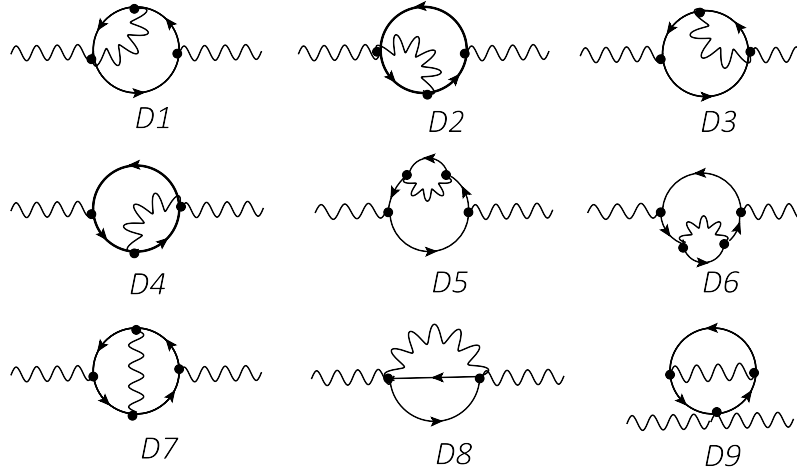


Figure 3: Two loop diagrams contributing to the two-point vertex function of the gauge superfield Γ_α .

which fixes the value of the Z_Φ counterterm as

$$Z_\Phi = 1 + \frac{i}{4(32\pi^2\epsilon)} \left[-(N+1)\lambda^2 + \frac{1}{4}(a+b)^2(2N+3)g^4 \right]. \quad (53)$$

Remembering Eq. (48), we see that Z_Φ depends on the gauge independent combination $a+b$. From this, it follows that the anomalous dimension γ_Φ will also be gauge independent.

The next step is to compute up to two loops the two-point vertex function of the gauge superfield Γ_α . The diagrams involved are represented in Figure 3, with the respective divergent contributions given in Table II. We verify that, for any gauge choice, all divergences

$D1$	$(3a - b) \{-p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$	$D2$	$(3a - b) \{-p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$
$D3$	$(3a - b) \{-p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$	$D4$	$(3a - b) \{-p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$
$D5$	$-2a \{p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$	$D6$	$-2a \{p_{\alpha\beta} + 3C_{\beta\alpha} D^2\}$
$D7$	$4 \{(4a + b)p_{\alpha\beta} + 3bC_{\beta\alpha} D^2\}$	$D8$	$-\{bp_{\alpha\beta} + 3aC_{\beta\alpha} D^2\}$
$D9$	0		

Table II: Divergent contributions from each diagram in Figure 3, omitting the common factor $\frac{1}{8} \left(\frac{N}{192\pi^2\epsilon} \right) i g^4 \int \frac{d^3p}{(2\pi)^3} d^2\theta \Gamma^\alpha(p, \theta) \Gamma^\beta(-p, \theta)$.

cancel among the graphs in Fig. 3. As a consequence, no infinite wave function renormalization of the CS superfield is needed, and therefore the anomalous dimension γ_{Γ} vanishes. This result extends for the massless matter case according to the Coleman-Hill theorem [41], and it was also verified in previous calculations performed in a specific gauge [15], as well as in the non-supersymmetric version of the model [42].

Finally, the evaluation of the divergent part of the four-point vertex function associated to the scalar superfield Φ , up to two loops, involve all diagrams in the Figure 4. The results are given in Table III, and lead to

$$S_{(\bar{\Phi}\Phi)^2}^{2loop} = \frac{i}{4(32\pi^2\epsilon)} \left[- (5N + 11) \lambda^3 - (a + b) \lambda^2 g^2 + \frac{1}{4} (a + b)^2 (2N + 5) \lambda g^4 \right. \\ \left. + \frac{1}{4} (a + b)^3 (N + 3) g^6 \right] \{ \delta_{im} \delta_{nj} + \delta_{jm} \delta_{ni} \} \int d^2\theta \Phi_n(0, \theta) \Phi_m(0, \theta) \bar{\Phi}_i(0, \theta) \bar{\Phi}_j(0, \theta) , \quad (54)$$

which implies that

$$Z_\lambda = 1 + \frac{1}{2(32\pi^2\epsilon)} \left[(5N + 11) \lambda^2 + (a + b) \lambda g^2 - \frac{1}{4} (a + b)^2 (2N + 5) g^4 \right. \\ \left. - \frac{1}{4} (a + b)^3 (N + 3) \lambda^{-1} g^6 \right] . \quad (55)$$

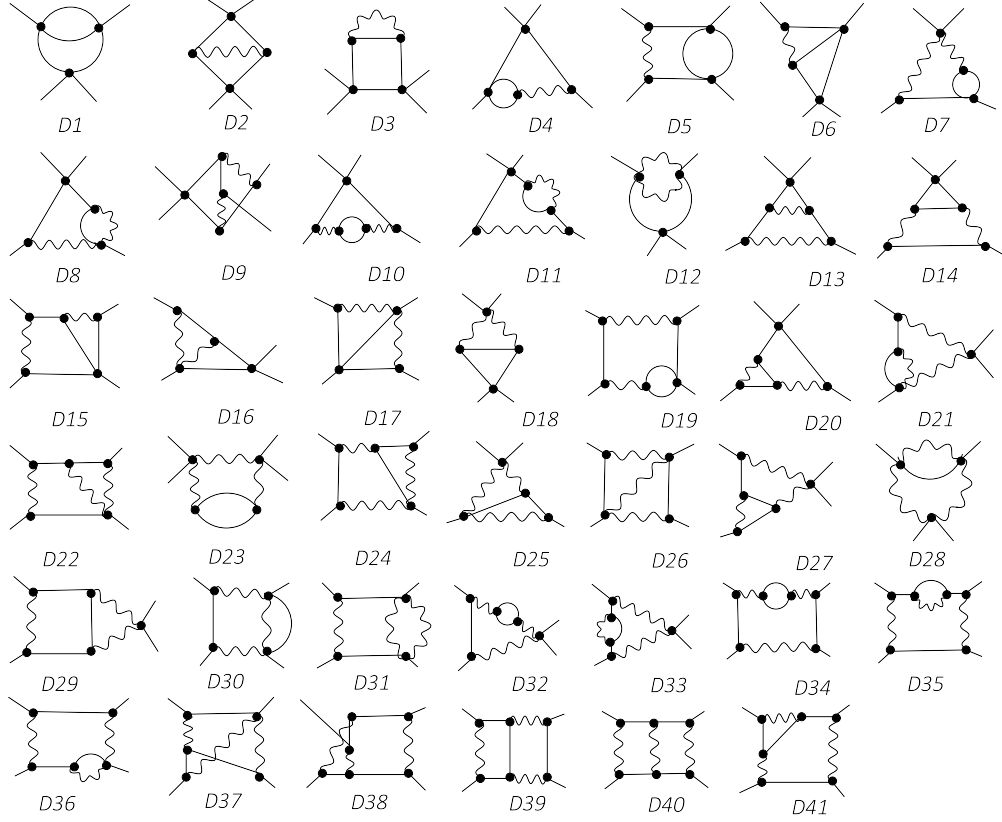


Figure 4: The complete set of two loops diagrams contributing to the four-point scalar vertex function.

In conclusion, all the renormalization constants Z_Φ , Z_Γ and Z_λ are independent of the choice of the gauge-fixing parameter. This same property will follow for the renormalization group functions that are calculated from these constants, and from the procedure described in [27], to the effective superpotential K_{eff} .

The explicit relations between bare and renormalized quantities are given by

$$\Gamma_0^\alpha = Z_\Gamma^{\frac{1}{2}} \Gamma^\alpha, \quad (56)$$

$$\Phi_0 = Z_\Phi^{\frac{1}{2}} \Phi, \quad (57)$$

$$\alpha_0 = \alpha Z_\Gamma, \quad (58)$$

$$g_0 = \mu^{\frac{\epsilon}{2}} g Z_\Gamma^{-\frac{1}{2}}, \quad (59)$$

$$\lambda_0 = \mu^\epsilon \lambda Z_\lambda Z_\Phi^{-2}, \quad (60)$$

where μ is a mass parameter introduced to keep g and λ dimensionless. From these, follow

$D1$	$-\frac{1}{4}(5N+11)\lambda^3$	$D2$	$\frac{1}{4}a(N+2)\lambda^2g^2$	$D3$	$-\frac{1}{4}a(N+4)\lambda^2g^2$
$D4$	0	$D5$	$-\frac{1}{4}(a-b)\lambda^2g^2$	$D6$	$\frac{1}{2}(a-b)\lambda^2g^2$
$D7$	0	$D8$	$\frac{1}{8}(b^2-4ab+3a^2)\lambda g^4$	$D9$	$\frac{3}{16}(a-b)^2\lambda g^4$
$D10$	0	$D11$	$\frac{1}{4}a(b-a)\lambda g^4$	$D12$	$\frac{3}{4}a^2\lambda g^4$
$D13$	$\frac{3}{4}a(a-b)\lambda g^4$	$D14$	0	$D15$	$-\frac{1}{4}(a-b)^2\lambda g^4$
$D16$	$\frac{3}{2}a(b-a)\lambda g^4$	$D16$	$\frac{3}{2}a(b-a)\lambda g^4$	$D17$	$\frac{1}{4}(a-b)^2\lambda g^4$
$D18$	$\frac{1}{8}(a+b)^2(1+N)\lambda g^4$	$D19$	0	$D20$	$-\frac{1}{8}(a-b)^2\lambda g^4$
$D21$	$-\frac{1}{8}(a-b)^2(3a-b)g^6$	$D22$	0	$D23$	$\frac{1}{16}(a+b)^3Ng^6$
$D24$	$\frac{1}{8}(a-b)^3g^6$	$D25$	$-\frac{1}{4}(a-b)(a^2+b^2)g^6$	$D26$	0
$D27$	$\frac{1}{8}(a-b)^3g^6$	$D28$	$\frac{1}{2}a(a^2+2b^2)g^6$	$D29$	$\frac{1}{16}(a-b)(a+b)^2g^6$
$D30$	$-\frac{1}{8}(a-b)^2(2a-b)g^6$	$D31$	0	$D32$	0
$D33$	$\frac{1}{8}a(a-b)^2g^6$	$D34$	0	$D35$	$-\frac{1}{8}a(a-b)^2g^6$
$D36$	$\frac{1}{8}(a-b)^2(3a-b)g^6$	$D37$	0	$D38$	0
$D39$	0	$D40$	0	$D41$	$\frac{1}{8}(b-a)^3g^6$

Table III: Divergent contributions arising from the diagrams presented in Figure 4; all contributions include the common factor $\frac{i}{32\pi^2\epsilon}\{\delta_{im}\delta_{nj}+\delta_{jm}\delta_{ni}\}\int d^2\theta\Phi_n(0,\theta)\Phi_m(0,\theta)\bar{\Phi}_i(0,\theta)\bar{\Phi}_j(0,\theta)$, where all external momenta were set to zero.

the renormalization group functions,

$$\gamma_\Gamma \equiv -\frac{\mu}{\Gamma}\frac{d}{d\mu}\Gamma = \frac{\mu}{2Z_\Gamma}\frac{d}{d\mu}Z_\Gamma, \quad (61)$$

$$\gamma_\Phi = \frac{\mu}{2Z_\Phi}\frac{d}{d\mu}Z_\Phi, \quad (62)$$

$$\beta_\alpha \equiv \mu\frac{d}{d\mu}\alpha = -2\alpha\gamma_\Gamma, \quad (63)$$

$$\beta_g = g\gamma_\Gamma, \quad (64)$$

$$\beta_\lambda = \epsilon\frac{\lambda^2}{Z_\lambda}\frac{\partial}{\partial\lambda}Z_\lambda + \frac{\epsilon\lambda g}{2Z_g}\frac{\partial}{\partial g}Z_g + 4\lambda\gamma_\Phi. \quad (65)$$

From these definitions, and the results presented in this section, we obtain

$$\gamma_\Phi = \frac{1}{4(32\pi^2)}\{(N+1)\lambda^2 - (2N+3)g^4\}, \quad (66)$$

$$\gamma_{\Gamma} = \beta_{\alpha} = \beta_g = 0, \quad (67)$$

and

$$\beta_{\lambda} = \frac{1}{16\pi^2} \{ 3 (N + 2) \lambda^3 + \lambda^2 g^2 - 2 (N + 2) \lambda g^4 - (N + 3) g^6 \}. \quad (68)$$

As stated before, these functions are independent of the gauge-fixing parameter. Our results agree with reference [15], except for overall numerical factors that arise due to their different definition of the scale μ . We stress however that in [15] the authors considered as true the gauge independence of these functions, a fact that has to be checked explicitly.

VI. CONCLUSION

The gauge independence of physical observables is an essential point to consider when studying gauge theories. In relation to the effective potential, which plays an essential role in the mechanism of symmetry breaking in many relevant models, the Nielsen identity is the key to understand how the effective potential can depend on the gauge choice, and yet physical quantities evaluated at its minima can be gauge independent.

In this paper we studied the Nielsen identity for a supersymmetric Chern-Simons model in the superfield formalism. After deriving the Nielsen identity in the superfield language, we argue that an explicit calculation of the complete effective superpotential V_{eff}^S , including any possible gauge dependence, is still a technically difficult task. As a first step in this direction, we consider the part of the effective superpotential which do not depend on supercovariant derivatives of the background scalar superfield, K_{eff} . It was already shown in [27] that K_{eff} can be calculated from the renormalization group functions, using the RGE. We then verify, by means of an explicit calculation, that the beta and gamma functions are gauge independent and, therefore, so is K_{eff} . Our results agree with the ones previously found in the literature, that were calculated in a specific gauge.

As a future perspective, we will try to extend these results to the full effective superpotential V_{eff}^S . This should be possible with the results given in this article, and same techniques used in [27], but generalized to the multiscale case [37, 38]. If these techniques are shown to be robust enough to prove that all terms in V_{eff}^S are fixed in terms of the coefficients of the

beta and gamma functions, we may be finally able to establish the gauge independence of the full effective superpotential V_{eff}^S .

Also, a generalization of this study for non Abelian models would be an interesting endeavor. Already at the perturbative level, in a non Abelian model the computation of the renormalization group functions would involve several additional diagrams, and even the ones already existent will be modified by group theoretical factors. So, it is not obvious that the dependence on the gauge parameter α , that cancelled among all diagrams in the Abelian case, as we shown, will also cancel in the non Abelian case. This would make the study of the Nielsen Identity much richer, since this identity would then be essential to prove the gauge invariance of the physical properties of the symmetry breaking. Finally, in non Abelian gauge theories there are also non perturbative aspects that are relevant, e.g., Gribov copies. These are profound questions that deserves more study.

Acknowledgments. This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), via the following grants: CNPq 482874/2013-9, FAPESP 2013/22079-8 and 2014/24672-0 (AFF), CAPES PhD grant (AGQ).

Two Loops Integrals

In obtaining the results in this paper, we have used the following two loops integrals in Minkowski spacetime, with spacetime metric $\eta^{\alpha\beta} \equiv \text{diag}(-, +, +)$, where $d^D l \equiv \mu^\epsilon d^{3-\epsilon} l$, $D = 3 - \epsilon$, and $C_{\alpha\beta}$ is the antisymmetric tensor used to lower and raise spinor indices [29].

$$\mathcal{I}_1 = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{1}{(k+p)^2 (q+k)^2 q^2} = -\frac{1}{32\pi^2 \epsilon}, \quad (69)$$

$$\mathcal{I}_2 = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\alpha\beta}}{(k+p)^2 (q+k)^2 q^2} = -\frac{p_{\alpha\beta}}{96\pi^2 \epsilon}, \quad (70)$$

$$\mathcal{I}_3 = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\alpha\beta}}{(k+p)^2 (q+k)^2 q^2} = \frac{p_{\alpha\beta}}{48\pi^2 \epsilon}, \quad (71)$$

$$\mathcal{I}_4 = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\alpha\beta} q_{\theta\lambda}}{(k+p)^2 k^2 (q+k)^2 q^2} = \frac{1}{192\pi^2\epsilon} (C_{\alpha\theta} C_{\beta\lambda} + C_{\beta\theta} C_{\alpha\lambda}) \quad (72)$$

$$\mathcal{I}_5 = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\mu\nu} k_{\theta\lambda}}{(k+p)^2 k^2 (k+q)^2 q^2} = -\frac{1}{96\pi^2\epsilon} (C_{\mu\theta} C_{\nu\lambda} + C_{\nu\theta} C_{\mu\lambda}), \quad (73)$$

$$\begin{aligned} \mathcal{I}_6 &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\lambda\theta} k_{\beta\rho}}{(k+p)^2 (p+q)^2 (k-q)^2 q^2} = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\lambda\theta} k_{\beta\rho}}{(k+p)^2 (p+q)^2 (k-q)^2 k^2} \\ &= -\frac{1}{192\pi^2\epsilon} (C_{\lambda\beta} C_{\theta\rho} + C_{\theta\beta} C_{\lambda\rho}), \end{aligned} \quad (74)$$

$$\begin{aligned} \mathcal{I}_7 &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\mu\beta} q_{\theta\lambda}}{(k+p)^2 (p+q)^2 (k-q)^2 q^2} = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\mu\beta} k_{\theta\lambda}}{(k+p)^2 (p+q)^2 (k-q)^2 k^2} \\ &= -\frac{1}{96\pi^2\epsilon} (C_{\mu\theta} C_{\beta\lambda} + C_{\beta\theta} C_{\mu\lambda}), \end{aligned} \quad (75)$$

$$\begin{aligned} \mathcal{I}_8 &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\mu\nu} q_{\lambda\theta} k_{\rho\sigma}}{(k+p)^2 (q+p)^2 (k-q)^2 q^2} = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\mu\nu} k_{\lambda\theta} q_{\rho\sigma}}{(k+p)^2 (q+p)^2 (k-q)^2 k^2} \\ &= \frac{1}{320\pi^2\epsilon} \left\{ (C_{\lambda\rho} C_{\theta\sigma} + C_{\theta\rho} C_{\lambda\sigma}) p_{\mu\nu} + (C_{\mu\rho} C_{\nu\sigma} + C_{\nu\rho} C_{\mu\sigma}) p_{\lambda\theta} + \frac{8}{3} (C_{\mu\lambda} C_{\nu\theta} + C_{\nu\lambda} C_{\mu\theta}) p_{\rho\sigma} \right\}, \end{aligned} \quad (76)$$

$$\begin{aligned} \mathcal{I}_9 &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{q_{\mu\lambda} k_{\zeta\theta} k_{\kappa\rho}}{(k+p)^2 k^2 (k+q)^2 q^2} = -\frac{1}{480\pi^2\epsilon} \{ (C_{\mu\zeta} C_{\lambda\theta} + C_{\lambda\zeta} C_{\mu\theta}) p_{\kappa\rho} \\ &\quad + (C_{\zeta\kappa} C_{\theta\rho} + C_{\theta\kappa} C_{\zeta\rho}) p_{\mu\lambda} + (C_{\mu\kappa} C_{\lambda\rho} + C_{\lambda\kappa} C_{\mu\rho}) p_{\zeta\theta} \}, \end{aligned} \quad (77)$$

$$\begin{aligned} \mathcal{I}_{10} &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\lambda\theta} k_{\mu\nu} k_{\kappa\rho} q_{\sigma\gamma}}{(k+p)^2 k^2 (k+q)^2 k^2 q^2} = \frac{1}{960\pi^2\epsilon} \{ (C_{\lambda\mu} C_{\theta\nu} + C_{\theta\mu} C_{\lambda\nu}) (C_{\kappa\sigma} C_{\rho\gamma} + C_{\rho\sigma} C_{\kappa\rho}) \\ &\quad + (C_{\lambda\kappa} C_{\theta\rho} + C_{\theta\kappa} C_{\lambda\rho}) (C_{\mu\sigma} C_{\nu\gamma} + C_{\nu\sigma} C_{\mu\gamma}) + (C_{\lambda\sigma} C_{\theta\gamma} + C_{\theta\sigma} C_{\lambda\gamma}) (C_{\mu\kappa} C_{\nu\rho} + C_{\nu\kappa} C_{\mu\rho}) \}, \end{aligned} \quad (78)$$

$$\begin{aligned} \mathcal{I}_{11} &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\delta\theta} k_{\sigma\rho} q_{\beta\gamma} q_{\alpha\lambda}}{(k+p)^2 (p+q)^2 (k-q)^2 k^2 q^2} = -\frac{1}{5(384\pi^2\epsilon)} \{ 6 (C_{\delta\sigma} C_{\theta\rho} + C_{\theta\sigma} C_{\delta\rho}) \times \\ &\quad (C_{\beta\alpha} C_{\gamma\lambda} + C_{\gamma\alpha} C_{\beta\lambda}) + (C_{\delta\beta} C_{\theta\gamma} + C_{\theta\beta} C_{\delta\gamma}) (C_{\sigma\alpha} C_{\rho\lambda} + C_{\rho\alpha} C_{\sigma\lambda}) \\ &\quad + (C_{\delta\alpha} C_{\theta\lambda} + C_{\theta\alpha} C_{\delta\lambda}) (C_{\sigma\beta} C_{\rho\gamma} + C_{\rho\beta} C_{\sigma\gamma}) \}, \end{aligned} \quad (79)$$

$$\begin{aligned}
\mathcal{I}_{12} &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\delta\theta} k_{\sigma\rho} q_{\beta\gamma} q_{\alpha\lambda}}{(k+p)^2 (q^2)^2 [(k-q)^2]^2} = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{k_{\delta\theta} k_{\sigma\rho} q_{\beta\gamma} q_{\alpha\lambda}}{(k^2)^2 (q+p)^2 [(k-q)^2]^2} \\
&= \frac{1}{4(160\pi^2\epsilon)} \left\{ -\frac{2}{3} (C_{\delta\sigma} C_{\theta\rho} + C_{\theta\sigma} C_{\delta\rho}) (C_{\beta\alpha} C_{\gamma\lambda} + C_{\gamma\alpha} C_{\beta\lambda}) + (C_{\delta\beta} C_{\theta\gamma} + C_{\theta\beta} C_{\delta\gamma}) \times \right. \\
&\quad \left. (C_{\sigma\alpha} C_{\rho\lambda} + C_{\rho\alpha} C_{\sigma\lambda}) + (C_{\delta\alpha} C_{\theta\lambda} + C_{\theta\alpha} C_{\delta\lambda}) (C_{\sigma\beta} C_{\rho\gamma} + C_{\rho\beta} C_{\sigma\gamma}) \right\}, \tag{80}
\end{aligned}$$

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